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Comprehensive Description of RESPONSE

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Abstract

This paper offers a comprehensive description of the integrated assessment model (IAM) RESPONSE developed at CIRED. RESPONSE aims at providing a consistent framework to appraise alternative modelling choices made by the main existing IAMs. It is designed as a flexible tool able to take different modelling structures in order to compare results from the modelling frameworks that have driven the so-called ``when flexibility'' controversy since the early 1990s dealing with the optimal timing of mitigation efforts and the optimal time profile of the social cost of carbon. RESPONSE is both sufficiently compact to be easily tractable and detailed enough to be as comprehensive as possible in order to capture a wide array of emblematic modelling choices, namely the forms of the damage function (quadratic vs. sigmoid) and the abatement cost (with or without inertia), the treatment of uncertainty, and the decision framework (one-shot vs. sequential).

Keywords: Integrated Assessment Model, Sequential Decision, Uncertainty, Inertia, Abatement Costs, Damage Function, Social Cost of Carbon.

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Description détaillée de RESPONSE

Résumé

Ce document offre une description détaillée du modèle intégré RESPONSE, développé au CIRED. RESPONSE offre un cadre de modélisation cohérent pour intégrer et évaluer les divers choix de modélisation faits par la majorité des modèles intégrés déjà existants. C'est un outil flexible, à même par exemple d'adopter et de comparer les différentes structures de modélisation qui sont discutées dans la controverse, ouverte dès le début des années 1990, sur le calendrier optimal de la mitigation et le profil temporel de la valeur sociale du carbone. RESPONSE est à la fois suffisamment compact pour être aisément manié, et assez détaillé pour représenter un large spectre de possibilités de modélisations : différentes formes de la fonction de dommages (quadratique ou sigmoïdale), de la fonction de coût d'abattement (avec ou sans inertie), de la représentation de l'incertitude, et du processus de décision (à une période ou séquentielle).

Mots-clés : modèle intégré économie-climat, décision séquentielle, incertitude, inertie, coûts d'abattement, fonction de dommages, valeur sociale du carbone.

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Abstract

This paper offers a comprehensive description of the integrated assessment model (IAM) RESPONSE developed at CIRED. RESPONSE aims at providing a consistent framework to appraise alternative modelling choices made by the main existing IAMs. It is designed as a flexible tool able to take different modelling structures in order to compare results from the modelling frameworks that have driven the so-called "when flexibility" controversy since the early 1990s dealing with the optimal timing of mitigation efforts and the optimal time profile of the social cost of carbon. RESPONSE is both sufficiently compact to be easily tractable and detailed enough to be as comprehensive as possible in order to capture a wide array of emblematic modelling choices, namely the forms of the damage function (quadratic vs. sigmoid) and the abatement cost (with or without inertia), the treatment of uncertainty, and the decision framework (one-shot vs. sequential).

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Active document: all internal references are hyperlinks to ease cross-checking.

Comprehensive Description of RESPONSE

Introduction

The main purpose of integrated assessment models (IAMs) is to provide policy options for climate change by combining the scientific and socio-economic aspects of the climate issue. As soon as 1996, the second assessment report by the Intergovernmental Panel on Climate Change (IPCC) included a review of the main existing IAMs at the time (Weyant et al., 1995) and defines their three core objectives: assessing climate change control policies, forcing multiple dimensions of the climate change problem into the same framework and quantifying the relative importance of climate change in the context of other environmental and non-environmental problems. Within this very large definition, a wide variety of models have been developed since the early 1990s.

RESPONSE is an IAM which provides a consistent framework to appraise alternative modelling choices made by the main existing IAMs. It has thus originally been designed as a flexible tool, able to adopt different modelling structures and compare results from the modelling frameworks that have driven the so-called “when flexibility” controversy since the early 1990s dealing with the optimal timing of mitigation efforts and the optimal time profile of the social cost of carbon. RESPONSE is both sufficiently compact to be easily tractable and detailed enough to be as comprehensive as possible in order to capture a wide array of emblematic modelling choices, namely the forms of the damage function (quadratic vs. sigmoid)(Ambrosi et al., 2003), of the abatement cost (with or without inertia)(Ha-Duong et al., 1997), the treatment of uncertainty (Manne and Richels, 1992; Nordhaus, 2008, 2011), and the decision framework (one-shot vs. sequential) (Chichilnisky and Heal, 1993; Kolstad, 1996; Ulph and Ulph, 1997; Ha-Duong, 1998; Goulder and Mathai, 2000; Pindyck, 2000).

This paper successively presents in section 1 the storyline of the model, its different modules and their equations in both the deterministic and uncertain case. A brief comparison of RESPONSE with other comparable IAMs is also provided in section 2. The first-order conditions, resolution of the optimization program, are given in section 3 while section 4 eventually defines the concept of social cost of carbon and presents its calculation method.

The GAMS code with a comprehensive description of the parametrization choices is available at `http://www.centre-cired.fr/spip.php?article1395`.

1 RESPONSE

1.1 Storyline of the model

RESPONSE is an IAM that couples a macroeconomic optimal growth model[1] with a simple climate model, following the tradition launched by the seminal DICE model (Nordhaus, 1994).

[1]Much like Ramsey-Cass-Koopmans' models (Ramsey, 1928; Koopmans, 1963; Cass, 1965).

The optimization program of RESPONSE aims at maximizing an intertemporal social welfare function composed of the consumption of a composite good. Greenhouse gases (GHG) are responsible for temperature increase and thus for climate damages. GHG emissions are a by-product of the production, offset by costly abatement effort. As climate damages negate part of the production, the optimization process consists in allocating the optimal share of the output among consumption, abatement and investment.

In the most basic modelling structure of RESPONSE, climate damage as well as abatement costs are represented with quadratic functions, as in DICE or PAGE (Stern, 2006; Hope, 2006). This gives a smooth increasing profile to both functions. The program is solved deterministically as no uncertainty on either techno-economic nor climate dynamics is taken into account.

The flexibility of the modelling structure of RESPONSE makes it possible to activate or deactivate some modelling options and thus for example to rebuild step by step the "when flexibility" controversy. It is possible to add an "inertia effect" in the abatement cost function to take into account the impact of the speed of abatement which can turn out to be critical in the case of very bad climate outcome that would require rapid change in abatement path. It is also possible to track non-linearity effects in climate damages, replacing the quadratic damage function with a sigmoid one which triggers a jump in damages from a certain level of temperature increase.

Eventually, RESPONSE enables us to switch from a deterministic to a uncertain model by integrating uncertainty on both climate sensitivity (and on atmospheric temperature increase) and climate damage[2]. The optimization program can be solved within both a one-shot and a sequential decision framework to appraise the impact of information arrival at different points in time t_i. At time t_i, uncertainties about climate sensitivity and damage are resolved[3]. This representation of uncertainty coupled with the inertia effect in abatement cost and non-linearity in climate damages allows us to address the discussion about the relative weight of economic and environmental irreversibilities and explore the concepts of "value of information" (Ambrosi et al., 2003) and "option value" of different types of climate policies (Pindyck, 2000; Ha-Duong, 1998).

1.2 The deterministic model

1.2.1 The representative household

We consider an infinite-horizon discrete-time economy inhabited by a continuum of size N_t of identical households. These households derive instantaneous utility from consumption of a composite good. They value their intertemporal utility:

$$V_{t_0} = \max_{a_t, C_t} \sum_{t=t_0}^{\infty} N_t \frac{1}{(1+\rho)^t} u\left(\frac{C_t}{N_t}\right), \tag{1.1}$$

with ρ the pure-time preference rate. The model does not endogenise the demographic dynamics, thus the number of households N_t evolves exogenously.

[2]We assume that both uncertainties are independent

[3]Note that RESPONSE does not address the learning issue (Kelly and Kolstad, 1999; Goulder and Mathai, 2000) and thus assumes that information arrives in an exogenous fashion.

We use a standard logarithmic utility function:

$$u(C_t) = \log(C_t) \tag{1.2}$$

This instantaneous utility function $u(C)$ has the standard properties: it is increasing in C, twice differentiable and concave. It furthermore follows the Inada condition $\lim\limits_{0^+} u' = +\infty$. The elasticity of intertemporal substitution is constant and equal to 1.

1.2.2 The production side

The economy produces a unique final good Y_t, from capital K_t and labour L_t. The production function is the traditional Cobb-Douglas:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1.3}$$

The share of capital in revenues is α. L_t is an exogenous variable representing the labour force; as there is no unemployment nor work-leisure trade-off, up to a normalization factor, it is equal to the number of households N_t. The total productivity factor A_t evolves exogenously.

Depending on consumption and abatement choices, the capital variable K_t evolves endogenously according to:

$$K_{t+1} = (1-\delta)K_t + Y_t \left[1 - C_a(a_t, a_{t-1}) - D(\theta_{A,t})\right] - C_t \tag{1.4}$$

The depreciation rate of the capital is δ. The abatement cost function $C_a(a_t, a_{t-1})$ depends on the abatement levels at the current period a_t and possibly of the past period a_{t-1}, in case of inertia. The damage function $D(\theta_{A,t})$ varies with the atmospheric temperature increase $\theta_{A,t}$. Abatement cost and damages are expressed relatively to total output Y_t, i.e. in percent of GDP.

Emissions of CO_2 are a by-product of the production, and can be offset by abatement effort a_t. Thus the total emission level is:

$$E_t = \sigma_t(1-a_t)Y_t \tag{1.5}$$

The carbon intensity of production σ_t is expected to decline progressively thanks to an exogenous technical progress:

$$\sigma_t = \sigma_0 \, \mathrm{e}^{-\psi_t t} \tag{1.6}$$

with $\psi_t > 0$ that captures the joint impact of technical change and depletion of fossil resources. If the economy grows at rate g, the level of carbon emissions is proportional to $\mathrm{e}^{(g-\psi_t)t}$. As long as $g > \psi_t$, carbon emissions would continue to grow over time. To guarantee that emissions decrease by the end of the century, as predicted by the overwhelming majority of available scenarios, ψ_t progressively increases so that it can become higher than g as follows, with $\beta > 0$:

$$\psi_t = \psi_0 \, \mathrm{e}^{-\beta t} + 1.1g(1 - \mathrm{e}^{-\beta t}), \tag{1.7}$$

Abatement a_t is expressed in fraction of emissions cut:

$$0 \leq a_t \leq 1 \tag{1.8}$$

If $a_t = 1$, then emissions become null, if $a_t = 0$, then no mitigation efforts are made.

1.2.3 Damage function

Two damage functions are used alternatively in RESPONSE. The first possibility is a quadratic function:

$$D(\theta_{A,t}) = \kappa\theta_{A,t}^2 \tag{1.9}$$

The second possibility is a sigmoid function (or logistic function):

$$D(\theta_{A,t}) = \kappa\theta_{A,t} + \frac{d}{1 + \mathrm{e}^{(\theta_D - \theta_{A,t})/\eta}} \tag{1.10}$$

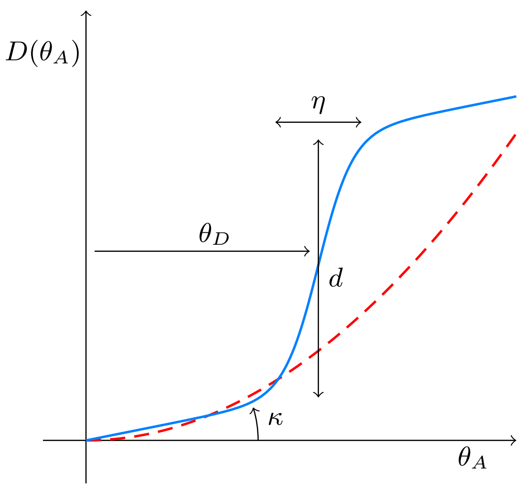
This damage function has a linear trend of slope κ with a smooth jump at a temperature threshold θ_D. The jump of size d is triggered when atmospheric temperature increase $\theta_{A,t}$ overshoots the threshold. Non-linearity in damages does not occur abruptly but instead progressively over a range η of temperature increase around θ_D.

One single function encapsulates these two forms of damage function:

$$D(\theta_{A,t}) = \kappa\theta_{A,t}^{1+\phi} + \frac{d}{1 + \mathrm{e}^{(\theta_D - \theta_{A,t})/\eta}} \tag{1.11}$$

with $\phi = 1$, $d = 0$ correspond to the quadratic case and $\phi = 0$, $d > 0$ to the sigmoidal case.

Figure 1: Possible forms of the damage function D in RESPONSE



The blue curve represents the sigmoidal case: θ_D is the temperature threshold where the non-linearity occurs, η is the width of the non-linearity phase, d is the size of the jump in damage during the non-linearity phase, and κ is a linear trend of damages. The red curve represents the quadratic case, κ symbolizing the curvature.

1.2.4 Abatement costs

The abatement cost function writes:

$$C_a(a_t, a_{t-1}) = PT_t \left(a_t\zeta + (BK - \zeta)\frac{(a_t)^\nu}{\nu} + \xi^2(a_t - a_{t-1})^2 \right) \tag{1.12}$$

The cost function has two main components: the absolute level of abatement $\frac{(a_t)^\nu}{\nu}$, with ν being a power coefficient, and a path-dependent function that penalizes the speed of decarbonisation $(a_t - a_{t-1})$. This abatement cost function allows us to account for an “inertia effect” (when $\xi \neq 0$) which penalizes abatement costs when the speed of abatement increases too rapidly. PT_t is a parameter of exogenous technical progress on abatement technologies, BK stands for the current price of the backstop technology or put in other words the marginal cost when abatement is total. ζ is the marginal cost of abatement when abatement is null.

1.2.5 The climate module

1.2.6 The three-reservoir climate module

The climate module is described through a three reservoirs linear carbon-cycle model. We use Nordhaus' carbon cycle (Nordhaus and Boyer, 2003), a linear three-reservoir model (atmosphere, biosphere and surface ocean, and deep ocean). Each reservoir is assumed to be homogeneous in the short run. It is also characterized by a residence time and mixing rates with the two other reservoirs in the long run. Carbon flows between reservoirs depend on constant transfer coefficients. GHG emissions (solely CO_2 here) accumulate in the atmosphere and are slowly removed by biosphere and ocean sinks.

The dynamics of carbon flows is given by:

$$\begin{pmatrix} A_{t+1} \\ B_{t+1} \\ O_{t+1} \end{pmatrix} = \mathbf{C}_{trans} \begin{pmatrix} A_t \\ B_t \\ O_t \end{pmatrix} + \begin{pmatrix} E_t \\ 0 \\ 0 \end{pmatrix} \tag{1.13}$$

A_t represents the carbon stock of the atmosphere at time t, B_t the carbon stock of the upper ocean and biosphere at time t and O_t the carbon stock of deep ocean at time t; $\mathbf{C}_{trans}$ is the transfer coefficient matrix. As there is no direct exchange between atmosphere and deep ocean, $c_{AO} = c_{OA} = 0$.

This carbon cycle model is mainly criticized because transfer coefficients are constant. First, they do not depend on the carbon stock of the reservoir, so that deep ocean reservoir never saturates. Actually, it is $CaCO_3$ neutralization and silicate weathering that restore the buffering capacity of the ocean, on a time scale of hundred of thousands of years (Archer and Brovkin, 2008; Archer et al., 2009). Second, they are not influenced by human activities such as deforestation which not only emits carbon dioxide, but also reduces the overall biosphere sinks (Gitz et al., 2003). Finally, possible positive feedbacks between ongoing climate change and the carbon cycle (Friedlingstein et al., 2006; Tol, 2009) are not taken into account.

1.2.7 Asymptotic behaviour of the carbon cycle model

We use Nordhaus' calibration of carbon-cycle models (for a ten-year time step):

$$C_{trans} = \begin{pmatrix} c_{AA} & c_{AB} & 0 \\ c_{BA} & c_{BB} & c_{BO} \\ 0 & c_{OB} & c_{OO} \end{pmatrix} = \begin{pmatrix} 0.66616 & 0.27607 & 0 \\ 0.33384 & 0.60897 & 0.00422 \\ 0 & 0.11496 & 0.99578 \end{pmatrix}$$

We derive the asymptotic behaviour of the climate model. First note that $\forall j \sum_i c_{ij} = 1$, so that carbon is just mixed among reservoir but does not disappear: $A_{t+1} + B_{t+1} + O_{t+1} = A_t + B_t + O_t + E_t$. So the total carbon stock at t is $A_{t_0} + B_{t_0} + O_{t_0} + \sum_{s=t_0}^{t-1} E_s$.

The transfer matrix has three eigenvalues, approximately 0.33, 0.94 and exactly 1, the eigenvector for the eigenvalue 1 is approximately:

$$e_1 = \begin{pmatrix} 0.02845 \\ 0.03440 \\ 0.93715 \end{pmatrix} \tag{1.14}$$

Let us note C_∞ the final carbon stock in all reservoir. One has $C_\infty = A_0 + B_0 + O_0 + \sum_{t=0}^{\infty} E_t$, the final carbon stock is the sum of the initial carbon stock plus the carbon released by emissions. The final state is collinear with the 1-eigenvector, it is $e_1 C_\infty$. With this model, only 2.8% of emissions stay in the atmosphere in the long run. Equilibrium between atmospheric reservoir and biosphere and upper ocean reservoir is established on a timescale of 50 years, whereas equilibrium with deep ocean is established over a time of $1,000$ years.

1.2.8 The two-box temperature module

The temperature module resembles Schneider and Thompson's two-box model (Schneider and Thompson, 1981) and builds on Ambrosi et al. (2003). Two equations are used to describe global mean temperature variation since pre-industrial times in response to additional GHG forcing. More precisely, the model describes the modification of the thermal equilibrium between the atmosphere and surface ocean in response to anthropogenic greenhouse effects.

The radiative forcing equation at time t is given by:

$$F(A_t) = F_{2x} \log_2(A_t / A_{PI}) \tag{1.15}$$

where F_{2x} is the instantaneous radiative forcing for a doubling of pre-industrial concentration; and A_{PI} is the atmospheric stock at pre-industrial times.

The temperature equation is given by:

$$\begin{pmatrix} \theta_{A,t+1} \\ \theta_{O,t+1} \end{pmatrix} = \begin{pmatrix} 1 - \sigma_1 (F_{2x}/\vartheta_{2x} + \sigma_2) & \sigma_1 \sigma_2 \\ \sigma_3 & 1 - \sigma_3 \end{pmatrix} \cdot \begin{pmatrix} \theta_{A,t} \\ \theta_{O,t} \end{pmatrix} + \begin{pmatrix} \sigma_1 F(A_t) \\ 0 \end{pmatrix} \tag{1.16}$$

where $\theta_{A,t}$ and $\theta_{O,t}$ are, respectively, global mean atmospheric and oceanic temperature increases from pre-industrial times (in Kelvin); σ_1, σ_2, and σ_3 are transfer coefficients, and ϑ_{2x} is the climate sensitivity, i.e. the ultimate temperature increase due to a doubling of pre-industrial level of atmospheric GHG concentration.

1.2.9 Asymptotic behaviour of the temperature model

Let us note $\mathbf{T}_{trans}$ the transfert matrix for temperature (for one year). With a given carbon stock A^∞, a stable state is a solution of $\theta = \mathbf{T}_{trans}\theta + (F(A^\infty), 0)$, that is $(\theta^\infty, \theta^\infty)$, with $\theta^\infty = \vartheta_{2x} \log_2(A^\infty/A_{PI})$. With a climate sensitivity $\vartheta_{2x} = 3$, the eigenvalues of the matrix are 0.724 and 0.987. Thus the equilibrium is established in a time scale of 300 years.

1.3 The model with uncertainty

1.3.1 The representation of uncertainties

Moving from the deterministic case to the uncertain case aims at taking into account current limitations of human knowledge about climate change. Even though the two most recent Intergovernmental Panel on Climate Change reports, the Stern Review, and the series of climate catastrophes over the past decade have already brought the "climate proof", all kinds of controversies are far from resolved, especially on the value of climate sensitivity and the extent of climate damages. Then, instead of single values, scientific results in the field rather provide ranges of reasonable values along with levels of confidence. As no decisive scientific argument has been brought so far to pick one state of the world rather than another, there are different competing beliefs in the climate debate about which state of the world will occur.

To encompass the entire range of scientific uncertainties about climate damage, we assume that there are states ω of nature, different with respect to climate sensitivity ϑ_{2x}^ω and the form of the damage function:

$$D^\omega(\theta_{A,t}) = \kappa^\omega \theta_{A,t}^{1+\phi} + \frac{d^\omega}{1 + \mathrm{e}^{(\theta_D^\omega - \theta_{A,t})/\eta}} \tag{1.17}$$

As climate change is basically a non-reproducible event, subjective distribution of probabilities are given over the possible states of the world considering that climate sensitivity and climate damage are independent. These distribution of probabilities account for the different competing beliefs in the climate debate and RESPONSE can be run for each belief.

We assume that there is a period t_i at which information about the true state of the world arrives. Then, in $t_i + 1$ people adapt their behavior to the new informations. They accelerate abatement in the case of "bad news" or relax their efforts in the case of "good news". The question each stakeholder must consider then becomes: what is the good trade-off between the economic risk of rapid abatement now (risk that premature capital stock retirement would later be proved unnecessary) against the corresponding risk of delay (risk that more rapid reduction would then be required, necessitating premature retirement of future capital stock)?

Such modelling of uncertainty makes it possible to appraise whether taking into account both kinds of uncertainties affects the solution by inducing more conservative (i.e. precautionary) decisions. This is particularly interesting when the damage function is not simply increasing but also non-linear, as it is the case with the sigmoid damage function.

A two-step analysis is conducted that mainly consists in solving the program recursively. The intertemporal optimization program is divided between two subprograms, after and before the information arrival date t_i respectively.Note that we can also account for the case with deep uncertainty when there is no resolution of information, if we take $t_i = \infty$.

1.3.2 After uncertainty is resolved

We first consider that the economy starts at time $t_i + 1$ when the true state of nature ω is known, that is, the climate sensitivity ϑ_{2x}^{ω} and the damage function D^{ω} are known. The intertemporal maximization program beginning at $t_i + 1$ is the same as in the deterministic case investigated previously. Variables corresponding to the solution of this program will be denoted by an upper script ω.

When we compute the discounted utility along the solution, we get the welfare $V_{t_i+1}(\omega)$ for whatever true value of the state of nature ω is revealed at t_i.

1.3.3 Before uncertainty is resolved

Before information arrival on the true states of nature at the end of period t_i, the objective function to maximize writes[4]:

$$W_{t_0} = \max_{\overline{a}_t, \overline{K}_t} \mathbb{E}\left[\sum_{t=t_0}^{t=t_i} \frac{1}{(1+\rho)^t} u(C_t^{\omega}) + V_{t_i+1}(\omega)\right]. \tag{1.18}$$

The variables set at a fixed level in all states of nature before t_i are overlined. This is the case for capital and abatement variables $\overline{K}_t$ and $\overline{a}_t$, and thus also for production $\overline{Y}_t$, emissions $\overline{E}_t$ and carbon stocks $\overline{A}_t$, $\overline{B}_t$, $\overline{O}_t$. The other variables which depend on the state of nature ω are written with an upper script. This is the case for the temperatures $\theta_{A,t}^{\omega}$, $\theta_{O,t}^{\omega}$ because their evolution depends on the unknown climate sensitivity ϑ_{2x}^{ω}. The damage function D^{ω} also depends on the state of nature. So does the consumption C^{ω} by equation (1.4). This implicitly means that different damages across different states of nature only affect consumption level and not the investment. If the consumption were also set at a fixed level, then the observation of either the investment or the capital would immediately lead to the observation of the true state of nature. This is why we make the hypothesis that the consumption cannot be observed per se, but instead that only the sum of the consumption and the damage can be quantified. We guess that it is easier to observe the level of investment than the consumption one and therefore that it makes sense to consider the capital variable as the control variable.

2 Comparing RESPONSE with standard IAMs

Due to the huge number of existing climate-economy models and the very large variety of their structures, we limit our comparison to five leading IAMs that have fuelled the "when flexibility" controversy. We thus concentrate on global and compact models which use welfare maximization and study in a way

[4]Here and in the rest of the document, $\mathbb{E}$ stands for the expectation operator: $\mathbb{E}[f] = \sum_{\omega} p(\omega) f(\omega)$.

or another the problem of uncertainty, either through a Monte-Carlo analysis or explicit probability distributions. We look in detail at DICE-2007 and PAGE-2006 and discuss more rapidly other IAMs in light of RESPONSE's modelling hypothesis.

2.1 RESPONSE and DICE

RESPONSE and DICE-2007 share the same perspective of neoclassical economic growth theory. In both models, "economies make investments in capital, education, and technologies, thereby abstaining from consumption today, in order to increase consumption in the future" while also "including the "natural capital" of the climate system as an additional type of "capital stock" (Nordhaus, 2007). Abatement policy choices involve a trade-off between consumption reduction today and potentially harmful climate change tomorrow. As in RESPONSE, DICE also "aggregates different countries into a single level of output, capital stock, technology, and emissions".

The most important modelling differences between the two IAMs however lie in the malleable specification of the damage and the abatement cost functional forms, and the possibility of introducing sequential decision-making[5], which makes RESPONSE able to account for a wider diversity of structural hypothesis. We summarize them in Table 1.

2.2 RESPONSE and PAGE

The spirit of the structural modelling choices of PAGE-2002 (Hope, 2006) is quite similar to DICE-2007 and to RESPONSE. There are however notable differences, especially in the treatment of uncertainty. In fact, DICE-2007 is purely deterministic while an explicit treatment of uncertainty can be introduced in RESPONSE. In PAGE-2002, "each uncertain input parameter (e.g., equilibrium warming from a doubling of CO_2 concentration) is represented by a probability distribution. PAGE-2002 has about eighty uncertain input parameters, the exact number depending on the regions and impact sectors used for a given run of the model. A full run of the PAGE-2002 model involves repeating the calculations of the following output variables: global warming over time, damages, adaptive costs, and preventative costs." The method used is the Latin Hypercube Sampling (LHS), which is supposedly more efficient than random "Monte Carlo". However, contrary to RESPONSE, the representation of uncertainty is built "outside" of the model and not per se.

The most important changes between DICE and PAGE are in the numerical hypothesis which are sometimes strongly opposed. In a sense, RESPONSE is thus a "meta-model" as DICE or PAGE only represent one of the several structural specifications it can embody.

2.3 RESPONSE and other IAMs

The DEMETER-1 model developed by van der Zwaan et al. (2002) focuses on the use of fossil-fuel versus non-fossil fuel energy sources. The production

[5]Among the numerous modified versions of DICE, some have however introduced certain forms of uncertainty.

Table 1: Main modelling differences between RESPONSE and DICE

DICE	RESPONSE
Constant relative risk aversion (CRRA) utility function $u(C_t) = N_t \frac{1}{1-\alpha}\left(\frac{C_t}{N_t}\right)^{1-\alpha}$	Logarithmic utility function $u(C_t) = N_t \log\left(\frac{C_t}{N_t}\right)$
Damages are proportional to world output and are polynomial functions of global mean temperature change $D(\theta_{A,t}) = \frac{1}{1+\pi_1\theta_{A,t}+\pi_2\theta_{A,t}^2}$	Highly malleable damage function which can take the polynomial form described in DICE but also a threshold form (or sigmoid) in order to represent potential non-linear effects $D(\theta_{A,t}) = \kappa(\theta_{A,t})^{1+\phi} + \frac{d}{1+\mathrm{e}^{(\theta_D-\theta_{A,t})/\eta}}$
"Abatement costs are proportional to global output and to a polynomial function of the reduction rate" $C_a(a_t) = \pi_t PT_t a_t^{\theta}$	Highly malleable abatement cost function: addition of a new term, representing a form of inertia in the abatement possibilities $C_a(a_t, a_{t-1}) = PT_t\Big(a_t\zeta$ $+(BK-\zeta)\frac{(a_t)^{\nu}}{\nu} + \xi^2(a_t-a_{t-1})^2\Big)$
Damage and abatement cost functions are multiplicatively limiting the output $Y_t D(\theta_{A,t})(1-C_a(a_t))$	Damage and abatement cost functions are additively limiting the level of production $Y_t\left(1-C_a(a_t,a_{t-1})-D(\theta_{A,t})\right)$
One-shot decision-making program	Possibility of sequential or one-shot decision-making program

function is thus of the following form:

$$Y_t = \left[A_t \left(K_t^{\alpha} L_t^{1-\alpha}\right)^{\gamma} + B_t \left(\left(F_t^{\chi} + N_t^{\chi}\right)^{\frac{1}{\chi}}\right)^{\gamma}\right]^{\frac{1}{\gamma}}$$

F_t and N_t respectively represent the fossil-fuel and non fossil-fuel inputs, considered as good substitutes. The constant elasticity of substitution (CES) function chosen for the energy part of the production function ensures that there is always some interest in investing in non-fossil fuel energy. In the most simple version of the model, the technological stock evolves exogenously both through A_t and B_t. Contrary to RESPONSE, DEMETER-1 thus explicitly represents the dichotomy between fossil-fuel and non-fossil-fuel energy systems. It can also in further developments partly endogenise technological change through learning-by-doing.

The MERGE-2005 model developed by Manne and Richels (2005) extends the damage function of DICE to include so-called "non-market damages" which cost a fixed amount of global GDP once the threshold of a 2.5℃ increase in temperature is reached. The intention is thus close to RESPONSE's malleable damage function when it takes the "sigmoid" form. But Manne and Richels attribute their damage parameters to "the literature" and explicitly admit that "the parameters of this loss function are highly speculative". One objective of RESPONSE is on the contrary to let the question of highly speculative values of parameters open by describing the map of possible outcomes without making any arbitrary choice between them. Contrary to RESPONSE also, MERGE-2005 does not have any kind of representation of uncertainty but rather relies on "best-guess" estimates of likely outcomes.

The FUND model developed by (Anthoff et al., 2009; Anthoff and Tol, 2010) divides the world into a number of regions, each of which has multiple damage functions corresponding to the different sectors. For example the interaction of climate change and the agriculture sector leads to three different types of impacts, depending on the rate of climate change, its absolute level and carbon dioxide fertilization impact. The FUND model implicitly captures adaptive capacity in the energy and health sectors by assuming wealthier nations are less vulnerable to climate impacts, and representing a knowledge accumulation factor which impacts the cost of mitigation. No backstop technology is assumed, which, according to Anthoff and Tol (2010), leads to higher costs than other top-down models for high emissions reductions. Other sectors include forestry, water sources, coastal protection, land use, ecosystems, and extreme weather damages. Per capita income is exogenous and scenario-driven. The model runs from 1950 until 3000 in time-steps of a year, with the hypothesis that steady state is reached after 2300. The utility function is implicit in FUND, through the multiple equations of costs of impacts of the different sectors in the different regions. This makes a full comparison with RESPONSE difficult, although the relative precision of the representation of the impacts of climate change in FUND is indeed a matter of inspiration.

3 First-Order Conditions Resolution

In this part, our calculations follow the two-step resolution method already described in part 1.3.

3.1 After uncertainty is resolved

After uncertainty is resolved, we know the state of nature ω. The Lagrangian writes:

$$\mathbf{L}^{\omega} = \sum_{t=t_i+1}^{\infty} N_t \frac{1}{(1+\rho)^t} u\left(\frac{C_t^{\omega}}{N_t}\right) \tag{3.1}$$

$$+ \sum_{t=t_i+1}^{\infty} (\lambda_{A,t}^{\omega}, \lambda_{B,t}^{\omega}, \lambda_{O,t}^{\omega}) \begin{pmatrix} A_{t+1}^{\omega} - (c_{AA}A_t^{\omega} + c_{AB}B_t^{\omega} + (1-a_t^{\omega})\sigma_t Y_t^{\omega}) \\ B_{t+1}^{\omega} - (c_{BA}A_t^{\omega} + c_{BB}B_t^{\omega} + c_{BO}O_t^{\omega}) \\ O_{t+1}^{\omega} - (c_{OB}B_t^{\omega} + c_{OO}O_t^{\omega}) \end{pmatrix}$$

$$+ \sum_{t=t_i+1}^{\infty} (\nu_{A,t}^{\omega}, \nu_{O,t}^{\omega}) \begin{pmatrix} \theta_{A,t+1}^{\omega} - ((1-\sigma_1(\frac{F_{2x}}{\vartheta_{2x}^{\omega}} + \sigma_2))\theta_{A,t}^{\omega} + \sigma_1\sigma_2\theta_{O,t}^{\omega} + \sigma_1 F(A_t^{\omega})) \\ \theta_{O,t+1}^{\omega} - (\sigma_3\theta_{A,t}^{\omega} + (1-\sigma_3)\theta_{O,t}^{\omega}) \end{pmatrix}$$

$$+ \sum_{t=t_i+1}^{\infty} \mu_t^{\omega}\big(- K_{t+1}^{\omega} + (1-\delta)K_t^{\omega} + Y_t^{\omega}\left[1 - C_a(a_t^{\omega}, a_{t-1}^{\omega}) - D^{\omega}(\theta_{A,t}^{\omega})\right] - C_t^{\omega}\big)$$

$$+ \sum_{t=t_i+1}^{\infty} \overline{\tau}_t^{\omega}.(1-a_t) + \underline{\tau}_t^{\omega}.a_t$$

The Lagrange multiplier attached to the capital constraint (1.4) is μ_t^{ω}; the Lagrange multipliers attached to the carbon cycle dynamics constraints (1.13) are $\lambda_{A,t}^{\omega}$, $\lambda_{B,t}^{\omega}$, and $\lambda_{O,t}^{\omega}$. The Lagrange multipliers attached to the temperature constraints (1.16) are $\nu_{A,t}^{\omega}$ and $\nu_{O,t}^{\omega}$. The Lagrange multipliers attached to inequality constraints (1.8) are $\overline{\tau}_t^{\omega}$ and $\underline{\tau}_t^{\omega}$.

At the beginning of the program, stock variables are inherited from the past, i.e. from the maximization program under uncertainty. Some do not depend on the state of nature:

$$A_{t_i+1}^{\omega} = \overline{A}_{t_i+1},\ B_{t_i+1}^{\omega} = \overline{B}_{t_i+1},\ O_{t_i+1}^{\omega} = \overline{O}_{t_i+1},\ K_{t_i+1}^{\omega} = \overline{K}_{t_i+1} \tag{3.2}$$

By convention, $a_{t_i}^{\omega} = \overline{a}_{t_i}$.

We calculate the first-order conditions with respect to the two fluxes variables: C_t^{ω} and a_t^{ω}, and to the six stock variables: K_t^{ω}, A_t^{ω}, B_t^{ω}, O_t^{ω}, $\theta_{A,t}^{\omega}$, and $\theta_{O,t}^{\omega}$. Recall also that stock variables at $t = t_i + 1$ are initial conditions, so we cannot derive first-order conditions for them at this stage. We get:

- For consumption, $\forall t \geq t_i + 1$:

$$\frac{\partial \mathbf{L}^{\omega}}{\partial C_t^{\omega}} = 0 \quad \Leftrightarrow$$

$$\mu_t^{\omega} = u'\left(\frac{C_t^{\omega}}{N_t}\right)\frac{1}{(1+\rho)^t} \tag{3.3}$$

  Then μ_t^{ω} is the discounted marginal utility.

- For the abatement capacity, $\forall t \geq t_i + 1$:

$$\frac{\partial \mathbf{L}^\omega}{\partial a_t^\omega} = 0 \quad \Leftrightarrow$$

$$\lambda_{A,t}^\omega \sigma_t = \mu_t^\omega \partial_1 C_a(a_t^\omega, a_{t-1}^\omega) + \mu_{t+1}^\omega \partial_2 C_a(a_{t+1}^\omega, a_t^\omega)\frac{Y_{t+1}^\omega}{Y_t^\omega} + \frac{\overline{\tau}_t^\omega - \underline{\tau}_t^\omega}{Y_t^\omega} \tag{3.4}$$

For $t = t_i + 1$, recall the conventional notation that $a_{t_i}^\omega = \overline{a}_{t_i}$. Recall that $\overline{\tau}_t^\omega > 0$ only when $a_t^\omega = 1$, and $\underline{\tau}_t^\omega > 0$ only when $a_t^\omega = 0$.

- For capital, $\forall t \geq t_i + 2$:

$$\frac{\partial \mathbf{L}^\omega}{\partial K_t^\omega} = 0 \quad \Leftrightarrow$$

$$\begin{aligned} &\partial_K Y_t^\omega \left(1 - \frac{\lambda_{A,t}^\omega (1 - a_t^\omega)\sigma_t}{\mu_t^\omega} - C_a(a_t^\omega, a_{t-1}^\omega) - D^\omega(\theta_{A,t}^\omega)\right) \\ &= (1+\rho)\frac{u'\left(\frac{C_{t-1}^\omega}{N_{t-1}}\right)}{u'\left(\frac{C_t^\omega}{N_t}\right)} - (1-\delta) \end{aligned} \tag{3.5}$$

- For the carbon stocks, $\forall t \geq t_i + 2$:

$$\frac{\partial \mathbf{L}^\omega}{\partial A_t^\omega} = 0 \quad \Leftrightarrow$$

$$\lambda_{A,t-1}^\omega = \lambda_{A,t}^\omega c_{AA} + \lambda_{B,t}^\omega c_{BA} + \nu_{A,t}^\omega \sigma_1 F'(A_t^\omega) \tag{3.6}$$

$$\frac{\partial \mathbf{L}^\omega}{\partial B_t^\omega} = 0 \quad \Leftrightarrow$$

$$\lambda_{B,t-1}^\omega = \lambda_{A,t}^\omega c_{AB} + \lambda_{B,t}^\omega c_{BB} + \lambda_{O,t}^\omega c_{OB} \tag{3.7}$$

$$\frac{\partial \mathbf{L}^\omega}{\partial O_t^\omega} = 0 \quad \Leftrightarrow$$

$$\lambda_{O,t-1}^\omega = \lambda_{B,t}^\omega c_{BO} + \lambda_{O,t}^\omega c_{OO} \tag{3.8}$$

- For the temperatures, $\forall t \geq t_i + 2$:

$$\frac{\partial \mathbf{L}^\omega}{\partial \theta_{A,t}^\omega} = 0 \quad \Leftrightarrow$$

$$\begin{aligned} \nu_{A,t-1}^\omega = \quad & \nu_{A,t}^\omega \left(1 - \sigma_1\left(\frac{F_{2x}}{\vartheta_{2x}^\omega} + \sigma_2\right)\right) \\ & + \nu_{O,t}^\omega \sigma_3 + \mu_t^\omega \partial_\theta D^\omega(\theta_{A,t}^\omega) Y_t^\omega \end{aligned} \tag{3.9}$$

$$\frac{\partial \mathbf{L}^\omega}{\partial \theta_{O,t}^\omega} = 0 \quad \Leftrightarrow$$

$$\nu_{O,t-1}^\omega = \nu_{A,t}^\omega \sigma_1 \sigma_2 + \nu_{O,t}^\omega (1 - \sigma_3) \tag{3.10}$$

3.2 Before uncertainty is resolved

The Lagrangian of the maximization program then equals the expectation over the possible states of nature of the sum of the objective function and a cluster of dynamic equations.

The Lagrangian writes:

$$\mathbf{L}_u = \sum_{t=t_0}^{t=t_i} \mathbb{E}\left[\frac{1}{(1+\rho)^t} u(C_t^\omega, S_t^\omega) + V(\omega)\right] \tag{3.11}$$

$$+ \sum_{t=t_0}^{t_i} (\lambda_{A,t}, \lambda_{B,t}, \lambda_{O,t}) \begin{pmatrix} \overline{A}_{t+1} - \left(c_{AA}\overline{A}_t + c_{AB}\overline{B}_t + (1-\overline{a}_t)\sigma_t \overline{Y}_t\right) \\ \overline{B}_{t+1} - (c_{BA}\overline{A}_t + c_{BB}\overline{B}_t + c_{BO}\overline{O}_t) \\ \overline{O}_{t+1} - (c_{OB}\overline{B}_t + c_{OO}\overline{O}_t) \end{pmatrix}$$

$$+ \sum_{t=t_0}^{t=t_i} \mathbb{E}\left[(\nu_{A,t}^\omega, \nu_{O,t}^\omega) \begin{pmatrix} \theta_{A,t+1}^\omega - ((1-\sigma_1(\frac{F_{2x}}{\vartheta_{2x}^\omega} + \sigma_2))\theta_{A,t}^\omega + \sigma_1\sigma_2\theta_{O,t}^\omega + \sigma_1 F(\overline{A}_t)) \\ \theta_{O,t+1}^\omega - (\sigma_3\theta_{A,t}^\omega + (1-\sigma_3)\theta_{O,t}^\omega) \end{pmatrix}\right]$$

$$+ \sum_{t=t_0}^{t=t_i} \mathbb{E}\left[\mu_t^\omega(-\overline{K}_{t+1} + (1-\delta)\overline{K}_t + Y_t^\omega\left[1 - C_a(a_t^\omega, a_{t-1}^\omega) - D^\omega(\theta_{A,t}^\omega)\right] - C_t^\omega)\right]$$

$$+ \sum_{t=t_0}^{t_i} \overline{\tau}_t.(1-\overline{a}_t) + \underline{\tau}_t.\overline{a}_t \tag{3.12}$$

$$\tag{3.13}$$

We calculate the first-order conditions with respect to all endogenous variables: fluxes variables C_t^ω and $\overline{a}_t$, and stock variables $\overline{K}_t$, $\overline{A}_t$, $\overline{B}_t$, $\overline{O}_t$, $\theta_{A,t}^\omega$, and $\theta_{O,t}^\omega$. The derivation will be specific for stock variables at t_i+1, and for the flux variable $\overline{a}_{t_i}$ due to the inertia in abatement cost, for we have to take into account their impact on $V(\omega)$. We get:

- For consumption, $\forall t \leq t_i$:

$$\frac{\partial \mathbf{L}^\omega}{\partial C_t^\omega} = 0 \quad \Leftrightarrow$$
$$\mu_t^\omega = u'\left(\frac{C_t^\omega}{N_t}\right)\frac{1}{(1+\rho)^t} \tag{3.14}$$

- For the abatement capacity, $\forall t < t_i$:

$$\frac{\partial \mathbf{L}_u}{\partial \overline{a}_t} = 0 \quad \Leftrightarrow$$
$$\begin{aligned}\lambda_{A,t}\sigma_t = \quad & \mathbb{E}\left[\mu_t^\omega\right]\partial_1 C_a(\overline{a}_t, \overline{a}_{t-1}) \\ & + \mathbb{E}\left[\mu_{t+1}^\omega\right]\partial_2 C_a(\overline{a}_{t+1}, \overline{a}_t)\frac{\overline{Y}_{t+1}}{\overline{Y}_t} + \frac{\overline{\tau}_t - \underline{\tau}_t}{\overline{Y}_t}\end{aligned} \tag{3.15}$$

For $t = t_i$, actions, which are decided at the beginning of the period, cannot take into account the information that arrives in this period:

$$\frac{\partial \mathbf{L}_u}{\partial \overline{a}_{t_i}} = 0 \quad \Leftrightarrow$$

$$\begin{aligned}\lambda_{A,t_i}\sigma_{t_i} = \quad & \mathbb{E}\left[\mu^\omega_{t_i}\right] \partial_1 C_a(\overline{a}_{t_i}, \overline{a}_{t_i-1}) \\ & + \mathbb{E}\left[\mu^\omega_{t_i+1}\partial_2 C_a(a^\omega_{t_i+1}, \overline{a}_{t_i})\right] \frac{\overline{Y}_{t_i+1}}{\overline{Y}_{t_i}} + \frac{\overline{\tau}_{t_i} - \underline{\tau}_{t_i}}{\overline{Y}_{t_i}}\end{aligned} \tag{3.16}$$

because $\frac{\partial V(\omega)}{\partial \overline{a}_{t_i}} = \frac{\partial L^\omega}{\partial \overline{a}_{t_i}} = \mu^\omega_{t_i+1}\partial_2 C_a(a^\omega_{t_i+1}, \overline{a}_{t_i})\overline{Y}_{t_i+1}$.

- For capital, $\forall t \leq t_i$:

$$\frac{\partial \mathbf{L}_u}{\partial \overline{K}_t} = 0 \quad \Leftrightarrow$$

$$\begin{aligned}&\partial_K Y_t \left(1 - \frac{\lambda_{A,t}(1-\overline{a}_t)\sigma_t}{\mathbb{E}[\mu^\omega_t]} - C_a(\overline{a}_t, \overline{a}_{t-1}) - \frac{\mathbb{E}[\mu^\omega_t D^\omega(\theta_{A,t})]}{\mathbb{E}\left[\mu^\omega_t\right]}\right) \\ &= \frac{\mathbb{E}\left[\mu^\omega_{t-1}\right]}{\mathbb{E}\left[\mu^\omega_t\right]} - (1-\delta)\end{aligned} \tag{3.17}$$

For $t = t_i + 1$,

$$\frac{\partial \mathbf{L}_u}{\partial \overline{K}_{t_i+1}} = 0 \quad \Leftrightarrow$$

$$\begin{aligned}&\partial_K Y_{t_i+1}\Bigg(1 - \frac{\mathbb{E}[\lambda^\omega_{A,t_i+1}(1-a^\omega_{t_i+1})\sigma_{t_i+1}]}{\mathbb{E}\left[\mu^\omega_{t_i+1}\right]} \\ &\qquad - \frac{\mathbb{E}[\mu^\omega_{t_i+1}C_a(a^\omega_{t_i+1}, \overline{a}_{t_i})]}{\mathbb{E}\left[\mu^\omega_{t_i+1}\right]} - \frac{\mathbb{E}[\mu^\omega_{t_i+1}D^\omega(\theta_{A,t_i+1})]}{\mathbb{E}\left[\mu^\omega_{t_i+1}\right]}\Bigg) \\ &= \frac{\mathbb{E}\left[\mu^\omega_{t_i}\right]}{\mathbb{E}\left[\mu^\omega_{t_i+1}\right]} - (1-\delta)\end{aligned} \tag{3.18}$$

because $\frac{\partial V(\omega)}{\partial \overline{K}_{t_i+1}} = \frac{\partial L^\omega}{\partial \overline{K}_{t_i+1}} = \mu^\omega_{t_i+1}(1 - \delta + \partial_K Y_{t_i+1}(1 - C_a(a^\omega_{t_i+1}, \overline{a}_{t_i}) - D^\omega(\theta_{A,t_i+1}))) - \lambda^\omega_{A,t_i+1}(1-a^\omega_{t_i+1})\sigma_{t_i+1}\partial_K Y_{t_i+1}$.

- For the atmospheric carbon multiplier, $\forall t \leq t_i$, the atmospheric carbon multiplier dynamic reads:

$$\frac{\partial \mathbf{L}_u}{\partial \overline{A}_t} = 0 \quad \Leftrightarrow$$

$$\lambda_{A,t-1} = \lambda_{A,t}c_{AA} + \lambda_{B,t}c_{BA} + \mathbb{E}\left[\nu^\omega_{A,t}\sigma_1 F'(\overline{A}_t)\right] \tag{3.19}$$

$$\frac{\partial \mathbf{L}_u}{\partial \overline{B}_t} = 0 \quad \Leftrightarrow$$

$$\lambda_{B,t-1} = \lambda_{A,t}c_{AB} + \lambda_{B,t}c_{BB} + \lambda_{O,t}c_{OB} \tag{3.20}$$

$$\frac{\partial \mathbf{L}_u}{\partial \overline{O}_t} = 0 \quad \Leftrightarrow$$
$$\lambda_{O,t-1} = \lambda_{B,t} c_{BO} + \lambda_{O,t} c_{OO} \tag{3.21}$$

For $t = t_i + 1$,

$$\frac{\partial \mathbf{L}_u}{\partial \overline{A}_{t_i+1}} = 0 \quad \Leftrightarrow$$
$$\lambda_{A,t_i} = \mathbb{E}\left[\lambda^{\omega}_{A,t_i+1} c_{AA} + \lambda^{\omega}_{B,t_i+1} c_{BA} + \nu^{\omega}_{A,t_i+1} \sigma_1 F'(\overline{A}_{t_i+1})\right] \tag{3.22}$$

$$\frac{\partial \mathbf{L}_u}{\partial \overline{B}_{t_i+1}} = 0 \quad \Leftrightarrow$$
$$\lambda_{B,t_i} = \mathbb{E}\left[\lambda^{\omega}_{A,t_i+1} c_{AB} + \lambda^{\omega}_{B,t_i+1} c_{BB} + \lambda^{\omega}_{O,t_i+1} c_{OB}\right] \tag{3.23}$$

$$\frac{\partial \mathbf{L}_u}{\partial \overline{O}_{t_i+1}} = 0 \quad \Leftrightarrow$$
$$\lambda_{O,t_i} = \mathbb{E}\left[\lambda^{\omega}_{B,t_i+1} c_{BO} + \lambda^{\omega}_{O,t_i+1} c_{OO}\right] \tag{3.24}$$

- For the temperature, $\forall t \leq t_i + 1$:

$$\frac{\partial \mathbf{L}_u}{\partial \theta^{\omega}_{A,t}} = 0 \quad \Leftrightarrow$$
$$\begin{aligned} \nu^{\omega}_{A,t-1} = \quad & \nu^{\omega}_{A,t} \left(1 - \sigma_1 \left(\frac{F_{2x}}{\vartheta^{\omega}_{2x}} + \sigma_2\right)\right) \\ & + \nu^{\omega}_{O,t} \sigma_3 + \mu^{\omega}_t \partial_\theta D^{\omega}(\theta^{\omega}_{A,t}) Y^{\omega}_t \end{aligned} \tag{3.25}$$

$$\frac{\partial \mathbf{L}_u}{\partial \theta^{\omega}_{O,t}} = 0 \quad \Leftrightarrow$$
$$\nu^{\omega}_{O,t-1} = \nu^{\omega}_{A,t} \sigma_1 \sigma_2 + \nu^{\omega}_{O,t}(1 - \sigma_3) \tag{3.26}$$

4 The Social Cost of Carbon

4.1 Theoretical Definition

The social cost of carbon (SCC) is "the additional damage caused by an additional ton of carbon emissions. In a dynamic framework, it is the discounted value of the change in the utility of consumption denominated in terms of current consumption" (Nordhaus, 2008).

4.2 Definition in RESPONSE

At time t, if there is an additional ton of carbon in the atmosphere, this will increase A_{t+1} by a unit and thus decrease the welfare from $t+1$. This variation of welfare is captured by $\partial W_{t+1}/\partial A_{t+1} = -\lambda_{A,t+1}c_{AA} - \lambda_{B,t+1}c_{BA} - \nu_{A,t+1}\sigma_1 F'(A_{t+1})$. Once on an optimal path, this is equal, thanks to (3.6) or (3.19), to $-\lambda_{A,t}$. So, on an optimal path, the social cost of carbon is also related to the abatement cost thanks to (3.4) or (3.15). The SCC has to be counted in current utility units.

More precisely, the equations for SCC at the different stages of the model are given below.

After uncertainty is resolved ($t \geq t_i + 1$), for each state of the world ω, the SCC is:

$$\begin{aligned} SCC_t^\omega &= \frac{\lambda_{A,t}^\omega}{\mu_t^\omega} \\ &= \frac{1}{\sigma_t}\left(\partial_1 C_a(a_t^\omega, a_{t-1}^\omega) + \frac{\mu_{t+1}^\omega}{\mu_t^\omega}\frac{Y_{t+1}^\omega}{Y_t^\omega}\partial_2 C_a(a_{t+1}^\omega, a_t^\omega) + \frac{\overline{\tau}_t^\omega - \underline{\tau}_t^\omega}{\mu_t^\omega Y_t^\omega}\right) \end{aligned} \tag{4.1}$$

For $t = t_i + 1$, this formula is rewritten as:

$$\begin{aligned} SCC_{t_i+1}^\omega = \frac{1}{\sigma_{t_i+1}}\Bigg(&\partial_1 C_a(a_{t_i+1}^\omega, \overline{a}_{t_i}) + \frac{\mu_{t_i+2}^\omega}{\mu_{t_i+1}^\omega}\frac{Y_{t_i+2}^\omega}{\overline{Y}_{t_i+1}}\partial_2 C_a(a_{t_i+2}^\omega, a_{t_i+1}^\omega) \\ &+ \frac{\overline{\tau}_{t_i+1}^\omega - \underline{\tau}_{t_i+1}^\omega}{\mu_{t_i+1}^\omega \overline{Y}_{t_i+1}}\Bigg) \end{aligned} \tag{4.2}$$

Before uncertainty is resolved, $\forall t \leq t_i$ the SCC is:

$$\begin{aligned} SCC_t =& \frac{\lambda_{A,t}}{\mathbb{E}[\mu_t^\omega]} \\ =& \frac{1}{\sigma_t}\left(\partial_1 C_a(\overline{a}_t, \overline{a}_{t-1}) + \frac{\mathbb{E}[\mu_{t+1}^\omega \partial_2 C_a(a_{t+1}^\omega, \overline{a}_t)]}{\mathbb{E}[\mu_t^\omega]}\frac{\overline{Y}_{t+1}}{\overline{Y}_t} + \frac{\overline{\tau}_t - \underline{\tau}_t}{\mathbb{E}[\mu_t^\omega]\overline{Y}_t}\right) \end{aligned} \tag{4.3}$$

For $t \leq t_i - 1$, the formula simplifies to:

$$SCC_t = \frac{1}{\sigma_t}\left(\partial_1 C_a(\overline{a}_t, \overline{a}_{t-1}) + \frac{\mathbb{E}[\mu_{t+1}^\omega]}{\mathbb{E}[\mu_t^\omega]}\frac{\overline{Y}_{t+1}}{\overline{Y}_t}\partial_2 C_a(\overline{a}_{t+1}, \overline{a}_t) + \frac{\overline{\tau}_t - \underline{\tau}_t}{\mathbb{E}[\mu_t^\omega]\overline{Y}_t}\right) \tag{4.4}$$

Comparing the formula in the uncertain case with the certain case, we can give an interpretation of the uncertain social cost in terms of the social cost if uncertainty had been resolved. The uncertain social cost corresponds to the mean of social cost in the different scenarios, averaged by utility in these scenarios (prices in different scenarios are not comparable, only utility units can be added, this is done by using μ_t^ω). So if we defined SCC_t^ω as previously (see the formulas after uncertainty is resolved), this interpretation of the social cost is tantamount to the formula: $SCC_t = \frac{\mathbb{E}[\mu_t^\omega SCC_t^\omega]}{\mathbb{E}[\mu_t^\omega]}$ before uncertainty is resolved.

4.3 Computation

To get the value of $SCC_t^\omega = \frac{\lambda_{A,t}^\omega}{\mu_t^\omega}$ we use the shadow prices associated to the concentration dynamics for $\lambda_{A,t}^\omega$ and the capital dynamics for μ_t^ω computed by GAMS.

Conclusion

The main value of RESPONSE rests on its malleability which makes it possible to compare different modelling structures within a unique framework and thus to address not only the "when" and "how much" controversies, but also more theoretical discussions such as the role of beliefs and worldviews in the building of a policy consensus. The malleability of the damage (quadratic vs. sigmoid) and the abatement cost (inertia or not) functions, the explicit representation of uncertainty (in climate sensitivity, parameters of the damage function), and the possibility of sequential decision-making (with a differing date of arrival of the information) are RESPONSE's main assets.

RESPONSE also offers the possibility of promising future development in order to address other key points of current debates.

First in its current version RESPONSE cannot yet represent any kind of endogenous technical change, either in the form of research and development or in the form of learning-by-doing (except if we admit that inertia in abatement costs can be considered as a form of learning). As noted by Stanton et al. (2009), "an unrealistic picture of fixed, predictable technological change, independent of public policy, is often assumed in IAMs". The remark applies to RESPONSE so far, but it has to be noted that modelling increasing returns in technologies seems easier in a more disaggregated model which would explicitly describe different technologies, as it is the case in FUND for example.

An other path for future research might be to overcome the simplistic way climate damages impact the economy. In fact, we assume that damages are directly paid by the representative agent through a consumption loss. Neither the capital stock, nor the overall productivity can be impacted, so that the economy does not really have any "memory" of the damages it incurs.

Finally, if the explicit representation of uncertainty is certainly a progress compared to most of the equivalent compact IAMs, a discussion on the form of the probability distributions, the uncertain parameters to be selected, and their combined impact on the final outcome remains to be carried out.

Outside these suggestions for further developments of RESPONSE, we must keep in mind the intrinsic limitations of all integrated assessment models, as underscored by Ackerman et al. (2009). These limitations should not lead us however to reject the method, but rather to be much more cautious in the analysis and exploitation of the results than economists may have been in the past. The usual criticisms against welfare economics put aside (about the ethical difficulty of choosing a pure-time preference rate, the problem of aggregating preferences across different individuals, and the psychological assumptions it assumes about the behaviour of human-beings), there still remains two major shadow points: IAMs cannot "predict the unpredictable" nor "price the priceless" but pretend in a way to do so. RESPONSE, as most other IAMs, should thus be considered as a powerful tool to discuss competing policy options and modelling choices, the most interesting part though being the dialogue it can generate with other areas of research.

References

Ackerman, F., S.J. DeCanio, R.B. Howarth, and K. Sheeran (2009) — Limitations of integrated assessment models of climate change, *Climatic change* ***95***(3), pp. 297–315.

Ambrosi, P., J.C. Hourcade, S. Hallegatte, F. Lecocq, P. Dumas, and M. Ha-Duong (2003) — Optimal control models and elicitation of attitudes towards climate damages, *Environmental Modeling and Assessment* ***8***(3), pp. 133–147.

Anthoff, D. and R.S.J. Tol (2010) — The Climate Framework for Uncertainty, Negotiation and Distribution (FUND), Technical Description, Version 3.3, URL http://www. fund-model. org.

Anthoff, D., R.S.J. Tol, and G.W. Yohe (2009) — Risk aversion, time preference, and the social cost of carbon, *Environmental Research Letters* ***4***, pp. 024002.

Archer, D. and V. Brovkin (2008) — The millennial atmospheric lifetime of anthropogenic CO_2, *Climatic Change* ***90***(3), pp. 283–297.

Archer, D., M. Eby, V. Brovkin, A. Ridgwell, L. Cao, U. Mikolajewicz, K. Caldeira, K. Matsumoto, G. Munhoven, A. Montenegro, and others (2009) — Atmospheric lifetime of fossil fuel carbon dioxide, *Annual Review of Earth and Planetary Sciences* ***37***, pp. 117–134.

Cass, D. (1965) — Optimum growth in an aggregative model of capital accumulation, *The Review of Economic Studies* ***32***(3), pp. 233–240.

Chichilnisky, G. and G. Heal (1993) — Global environmental risks, *The Journal of Economic Perspectives* ***7***(4), pp. 65–86.

Friedlingstein, P., P. Cox, R. Betts, L. Bopp, W. Von Bloh, V. Brovkin, P. Cadule, S. Doney, M. Eby, I. Fung, and others (2006) — Climate-carbon cycle feedback analysis: Results from the C4MIP model intercomparison, *Journal of Climate* ***19***(14), pp. 3337–3353.

Gitz, V., P. Ciais, and others (2003) — Amplifying effects of land-use change on future atmospheric CO2 levels, *Global Biogeochemical Cycles* ***17***(1), pp. 1024–1029.

Goulder, L.H. and K. Mathai (2000) — Optimal CO2 abatement in the presence of induced technological change, *Journal of Environmental Economics and Management* ***39***(1), pp. 1–38.

Ha-Duong, M. (1998) — Quasi-option value and climate policy choices, *Energy Economics* ***20***(5-6), pp. 599–620.

Ha-Duong, M., M.J. Grubb, and J.C. Hourcade (1997) — Influence of socioeconomic inertia and uncertainty on optimal CO2-emission abatement, *Nature* ***390***(6657), pp. 270–273.

Hope, C. (2006) — The marginal impact of CO2 from PAGE2002: An integrated assessment model incorporating the IPCC's five reasons for concern, *Integrated assessment* ***6***(1), pp. 19–56.

Kelly, D.L. and C.D. Kolstad (1999) — Bayesian learning, growth, and pollution, *Journal of Economic Dynamics and Control* ***23***(4), pp. 491–518.

Kolstad, C.D. (1996) — Fundamental irreversibilities in stock externalities, *Journal of Public Economics* ***60***(2), pp. 221–233.

Koopmans, T.C. (1963) — Appendix to'On the Concept of Optimal Economic Growth', Cowles Foundation Discussion Papers.

Manne, A.S. and R.G. Richels (1992) — *Buying greenhouse insurance: the economic costs of carbon dioxide emission limits*, The MIT Press.

Manne, Alan and Richard Richels (2005) — Merge: An Integrated Assessment Model for Global Climate Change, in R. Loulou, J.-P. Waaub, and G. Zaccour (Eds.), *Energy and Environment*, pp. 175–189. Springer US.

Nordhaus, W.D. (1994) — *Managing the global commons: the economics of climate change*, Cambridge, MA: MIT Press.

Nordhaus, W.D. (2007) — A Review of the" Stern Review on the Economics of Climate Change", *Journal of Economic Literature* ***45***(3), pp. 686–702.

Nordhaus, W.D. (2008) — *A question of balance*, Yale University Press.

Nordhaus, W. (2011, October) — Estimates of the Social Cost of Carbon: Background and Results from tje RICE-2011 Model, Cowles Foundation for Research in Economics.

Nordhaus, W.D. and J. Boyer (2003) — *Warming the world: economic models of global warming*, the MIT Press.

Pindyck, R.S. (2000) — Irreversibilities and the timing of environmental policy, *Resource and energy economics* ***22***(3), pp. 233–259.

Ramsey, F.P. (1928) — A mathematical theory of saving, *The Economic Journal* ***38***(152), pp. 543–559.

Schneider, S.H. and S.L. Thompson (1981) — Atmospheric CO2 and climate: importance of the transient response, *Journal of Geophysical Research* ***86***(C4), pp. 3135–3147.

Stanton, E.A., F. Ackerman, and S. Kartha (2009) — Inside the integrated assessment models: four issues in climate economics, *Climate and Development* ***1***(2), pp. 166–184.

Stern, N. (2006) — *The Economics of Climate Change. The Stern Review*, Cambridge University Press.

Tol, Richard S. J. (2009, April) — Climate Feedbacks on the Terrestrial Biosphere and the Economics of Climate Policy: An Application of Fund, Technical Report WP288, Economic and Social Research Institute (ESRI).

ULPH, A. and D. ULPH (1997) — Global warming, irreversibility and learning, *The Economic Journal* ***107***(442), pp. 636–650.

VAN DER ZWAAN, B., R. GERLAGH, G. KLAASSEN, and L. SCHRATTENHOLZER (2002) — Endogenous technological change in climate change modelling, *Energy Economics* ***24***(1), pp. 1–19.

WEYANT, J. P., O. DAVIDSON, H. DOWLATABADI, J. A. EDMONDS, M. J. GRUBB, E. A. PARSON, R. G. RICHELS, J. ROTMANS, P. R. SHUKLA, R. S. J. TOL, W. R. CLINE, and S. FRANKHAUSER (1995) — Integrated Assessment on Climate Change: An Overview and Comparison of Approaches and Results, in P. BRUCE, H. LEE, AND E. HAITES (Eds.), *Climate change 1995. Economic and social dimensions of climate change. Contribution of Working Group III to the Second Assessment Report of the Intergovernmental Panel on Climate Change.*, pp. 367–396. Cambridge University Press.